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equations (4) giving the direction cosines of the remaining line, viz.,

$$\begin{cases} l_3 = \dfrac{tl_1m_1 \pm sm_1^2 - sl_1^2n_1 \pm tl_1m_1n_1}{l_1^2 + m_1^2}, \\ m_3 = \dfrac{- sl_1m_1n_1 \pm tm_1^2n_1 - tl_1^2 \mp sl_1m_1}{l_1^2 + m_1^2}, \\ n_3 = sl_1 \mp tm_1. \end{cases}$$

The solution last given is the most general rational solution possible; for evidently any other solution must correspond to a solution of equation (3) more general than (5). To show that (5) is the most general solution, let us simplify the notation and consider the most general rational solution, for x and y, of the equation

$$x^2 + y^2 = a^2 + b^2, \qquad \text{or} \qquad x^2 - a^2 = - y^2 + b^2.$$

Then

$$x + a = \frac{1}{k}(y - b), \qquad x - a = - k(y + b),$$

or

$$x = \frac{(1 - k^2)a - 2kb}{1 + k^2}, \qquad y = \frac{2ka + (1 - k^2)b}{1 + k^2}.$$

Hence we have determined the most general set of these mutually perpendicular lines each of which has rational direction cosines.

A NOTE ON THE PROBLEM OF THE EIGHT QUEENS.

By W. H. BUSSEY, University of Minnesota.

Finite geometries were defined by Veblen and Bussey in the *Transactions of the American Mathematical Society*, volume 7 (1906), pp. 241–259. References to existing literature of the subject were given in this MONTHLY, *1921*, 85–86. The simplest case of a finite plane geometry based upon an odd prime, the euclidean plane geometry, modulo 3, was presented in detail by Bennett, in this MONTHLY, *1920*, 357–361.

The Problem of the Eight Queens is the determination of the number of ways in which eight queens can be placed on a chess board—or, more generally, in which n queens can be placed on a square board of n^2 cells—so that no queen can take any other. It was proposed originally by Franz Nauck.[1]

The object of this note is to show that in the special case in which n is a prime number p there is a connection between the problem of the queens and the lines of the finite plane geometry of p points to the line.

The cells of the chess board are represented by their middle points which

[1] For the history of the problem see Ahrens, *Mathematische Unterhaltungen und Spiele*, Leipzig, 1901, chapter 9. A brief discussion of the problem and its solution is given by Ball, *Mathematical Recreations and Essays*, fifth, sixth or seventh edition, pp. 113–118.

constitute a finite euclidean plane geometry of p points to the line. In order that one queen may take another, the two queens must occupy cells whose representative points are on a vertical line, on a horizontal line, on a line of slope 1, or on a line of slope $p - 1$. (The slope $p - 1$ is the same as the slope -1 since $p - 1 \equiv -1$, modulo p.)

Therefore, if p queens be placed on the cells whose representatives are the points of a line whose slope is any one of the integers $2, 3, 4, \cdots, p - 2$, no queen can take any other queen. The total number of such lines, each of which furnishes a solution of the problem of the p queens, is $p^2 + p - 4p$ or $p^2 - 3p$. When $p = 5$, the number of solutions obtained by this method is 10, which happens to be all the solutions that exist. When $p = 7$, the number of solutions furnished by this finite geometry method is 28; but, as a matter of fact, there are 40 solutions[1] when $p = 7$. For higher values of p, the lines of the finite geometry furnish some, but not all, of the solutions of the problem.

AMONG MY AUTOGRAPHS.

By DAVID EUGENE SMITH, Columbia University.

25. Montucla's Closing Years.

The little that we know about the great men of our varied spheres of interest and the careless surmises that we make as to their lives are evident whenever we look even slightly below the surface that lies open to the world. Anyone who reads an ordinary biographical sketch of Montucla, for example, gains an impression of a man who was born, who wrote the first great history of mathematics, and who died in the fullness of years.[2] In an earlier article in this series[3] I have called attention to a little side-light thrown upon his life by a note from his collaborator Lalande. It seems worth the effort, however, to call further attention to his closing years by publishing a portion of a letter, now in my collection and written to some unnamed friend, which gives a nearer view of these last years of one whose portraits show as a well-fed "gentleman of the old school," bland, placid, content with the world, and appreciative of the praise that this same world had bestowed upon him.

The letter is garrulous to the point of being wearisome, but a portion will suffice to give a picture of a poor, harassed, discouraged old man, suffering in mind and body, neglected by his friends and deserted by his family,—a subject for the pity of a world not then given to pitying anyone, and of a later world

[1] Cf. Ball, *l.c.*, p. 116.

[2] Jean Étienne Montucla, born September 5, 1725; died at Paris, December 18, 1799. He was married at Grenoble in 1763. The first edition of his history appeared in 1758. It is a work of great impartiality and erudition. By the Revolution he lost his position in the civil service and was left with no means of livelihood. In 1794 he secured a place under the revolutionary government and he finally, four months before his death, secured a small pension of 2,400 francs.

[3] This Monthly, *1921*, 207.